IV. Conclusions

A new method has been presented for computing the resonant frequencies of cylindrical dielectric resonators.

Although the method by Konishi *et al.* [2] is more accurate, particularly for resonators with lower dielectric constants, their method is considerably more complicated than the present one. The method in [4] is also more complicated than the present one. With almost the same order of simplicity in formulation and computational labor as the magnetic wall model [1], the present method provides results in close agreement with data reported in [2].

Acknowledgment

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Conversion Loss Limitations on Schottky-Barrier Mixers

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***Abstract*—A new set of criteria involving diode area, material parameters, and temperature is introduced for the Schottky-barrier mixer diode that must be considered if its usage is to be extended to the submillimeter wavelength region or cryogenically cooled to reduce the noise contribution of the mixer. It has been well established that, in order to reduce the parasitic loss as the frequency is increased, it is necessary to reduce the area of the diode. What has not been analyzed heretofore is the effect that a reduction in diode area can have on the intrinsic conversion loss L_0 of the diode resulting from its nonlinear resistance. This analysis focuses on the competing requirements of impedance matching the diode to its imbedding circuit and the finite dynamic range of the nonlinear resistance. As a result, L_0 can increase rapidly as the area is reduced. Results are first expressed in terms of dimensionless parameters, and then some respresentative examples are investigated in detail. The following conclusions are drawn: a large Richardson constant extends the usefulness of the diode to smaller diameters, and hence, shorter wavelengths; cooling a thermionic emitting diode can have a very detrimental effect on L_0; impedance mismatching is found, in general, to be a necessity for minimum conversion loss; and large barrier heights are desirable for efficient tunnel emitter converters.**

I. Introduction

The metal–semiconductor contact, or Schottky-barrier diode, has a long history of utilization as a mixer element [1], [2]. Its use has progressed to higher and higher frequencies, with the highest frequency recently being demonstrated by Fetterman *et al.*, who observed mixing at 3 THz with a GaAs Schottky diode [3]. For efficient operation at submillimeter wavelengths, many previously accepted tenets applicable to the design of microwave Schottky diodes must be reexamined.

Mixer conversion loss L_c, defined as the ratio of available power from the RF source to the power absorbed in the IF load, can be expressed in the form

$$L_c = L_0 L_p. \tag{1}$$

The intrinsic conversion loss L_0 is the loss arising from the conversion process within the nonlinear resistance of the diode and includes the impedance mismatch losses at the RF and IF ports. The parasitic loss L_p is the loss associated with the parasitic elements of the diode, the junction capacitance, and spreading resistance. Defined as the ratio of total power absorbed by the impedance R_m of the nonlinear resistance at the signal frequency, L_p is given by [4]

$$L_p = 1 + R_s/R_m + \omega_1^2 C^2 R_m R_s \tag{2}$$

where ω_1 is the signal angular frequency and C is the junction capacitance. The spreading resistance R_s is the resistance resulting from constriction of current flow in the semiconductor near the contact and is in series with the parallel elements C and R_m. Since $C \propto d^2$ and $R_s \propto d^{-1}$, where d is the diameter of the junction, (2) indicates that d should be reduced as the frequency of interest is increased. With the development of electron beam fabrication techniques [5], [6], the ability to produce Schottky barriers with dimensions of the order of a few hundred angstroms is imminent. However, the effect of a reduction in area on the intrinsic conversion loss L_0 must also be evaluated to determine overall mixer performance. This consideration is the central topic of this short paper.[1]

The dependence of L_0 on area originates in the impedance requirements the circuit places on the device. In order for the diode, driven by a local oscillator (LO), to couple most efficiently to a circuit with a specified impedance, it must pass approximately the same current, independent of the junction size. Hence reducing the size of the diode increases the current density through the device and, as a consequence, the dc bias voltage V_0 must be increased. Increasing V_0 limits the useful amplitude of the LO voltage V_1 because the current–voltage (I–V) characteristic of the junction in the forward direction is only nonlinear for applied voltages less than the barrier height potential V_B of the metal–semiconductor interface. Since $V_0 + V_1 \leq V_B$, decreasing the area serves to limit V_1, and consequently may increase L_0.

Because of the inverse relationship between the RF impedance of the diode and the bias current, superior results should be obtained for small areas if the Richardson constant of the semiconductor and the impedance of the circuit are large. The much larger Richardson constant of silicon extends its usefulness to smaller diameters than gallium arsenide. Moreover, it is predicted that the diode should be operated in an impedance mismatched condition; cooling a thermionic emitting diode can have a very detrimental effect on L_0, and large values of barrier height are desirable for efficient tunnel emitter converters.

From the classical conversion loss equations developed in Section II, specific situations are analyzed in Section III. Optimum coupling between the diode and the circuit is first analyzed. This result is applied to both thermionic emitting n-GaAs and n-Si Schottky diodes operating at 290 and 77 K,

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[1] For examples of L_p values with Schottky barriers on GaAs, Si, and Ge for wavelengths extending into the submillimeter, the reader is referred to [7].

TABLE I
LIST OF IMPORTANT SYMBOLS

L_0	Intrinsic conversion loss
L_{00}	Intrinsic conversion loss with optimum coupling
L_p	Parasitic loss
R_m	Barrier resistance at ω_1 with local oscillator
C	Junction capacitance
R_s	Spreading resistance
ω, ω_1, ω_2	Local oscillator, RF, and IF angular frequencies
i	Diode current
i_0	Preexponential current term
V	Voltage applied to the diode
V_0	DC bias voltage
V_1	Local oscillator voltage amplitude
g(t)	Diode conductance with applied local oscillator
g_0, g_1, g_2	Fourier series coefficients of g(t)
I_0, I_1, I_2	Modified Bessel functions of the first kind
R_1, R_2	RF source and IF load impedances
R_{10}, R_{20}	RF source and IF load impedances for optimum coupling
y_1, y_2	Impedance mismatch ratios
d	Junction diameter
d_m	Smallest junction diameter for which $y_1 = y_2 = 1$
J_{max}	Current density at flat-band voltage
V_b	Barrier height potential (flat-band voltage)
ε	Permittivity of semiconductor
N	Majority carrier concentration
A^*	Richardson constant
q	Electronic charge
k	Boltzmann's constant
T	Temperature (K)
$\hbar$	$(\frac{1}{2\pi})$ x Planck's constant

and to a tunnel emitting n-GaAs Schottky. The effect of impedance mismatching is then explored.

II. Classical Formulation

The I–V characteristic of a Schottky-barrier diode can be written in the form [8], [9]

$$i = i_0[\exp(SV) - 1], \qquad V < V_B. \tag{3}$$

Material parameters and the temperature of the junction determine the emission mechanism responsible for conduction and, consequently, determine i_0 and S in (3). The effects of these parameters on conversion are examined in subsequent sections, but first the basic mixing equations will be outlined. The symbols used are defined in Table I.

The broad-band Y-connected [10] circuit shown in Fig. 1 and the classical analysis found in Torrey and Whitmer [1] is employed. The crux of the analysis is the treatment of the conductance waveform of the nonlinear resistance. This conductance is a periodic function whose period is determined by the large LO voltage. As such, the conductance $g(t)$ can be expanded into a Fourier series

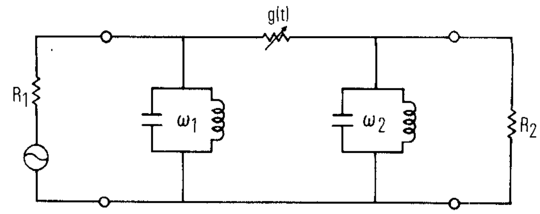

Fig. 1. A mixer in a Y-connected circuit with an RF source impedance R_1 and an IF load impedance R_2. The L-C parallel circuits are tuned to $\omega_{1,2}$ and are assumed to be open circuited at $\omega_{1,2}$ and short circuited at all other frequencies. In the broad-band mixer, the bandwidth of the input L-C circuit has sufficient width to include both the signal and image frequencies.

$$g(t) = \tfrac{1}{2}g_0 + g_1 \cos \omega t + g_2 \cos 2\omega t + \cdots \tag{4}$$

where ω is the LO angular frequency.

Conventional small-signal analysis of the circuit then yields

$$L_0 = \frac{1}{2y_1y_2\eta}[y_1 + y_2 + (y_1y_2 + 1)\sqrt{1-\eta}]^2 \tag{5}$$

where

$$y_1 = R_{10}/R_1 \tag{6}$$

$$y_2 = R_{20}/R_2 \tag{7}$$

$$\eta = \frac{2g_1^2}{g_0(g_0 + g_2)} \tag{8}$$

$$R_{10} = \frac{2/g_0}{(1 + g_2/g_0)\sqrt{1-\eta}} \tag{9}$$

$$R_{20} = \frac{2/g_0}{\sqrt{1-\eta}}. \tag{10}$$

Minimization of L_0 with respect to the parameters y_1 and y_2 yields optimum conditions of $y_1 = y_2 = 1$. Therefore, the conversion loss with optimum coupling L_{00} becomes from (5)

$$L_{00} = \frac{2}{\eta}(1 + \sqrt{1-\eta})^2. \tag{11}$$

The significance of this equation is discussed further in Section III.

With a dc bias V_0 and a sinusoidal LO voltage $V_1 \cos \omega t$ applied to the junction, the conductance can be expanded into a series of modified Bessel functions of the first kind $[I_n(x) = j^{-n}J_n(jx)]$ yielding

$$g_0 = 2Si_0I_0(SV_1)\exp(SV_0) \tag{12}$$

$$g_1/g_0 = I_1(SV_1)/I_0(SV_1) \tag{13}$$

$$g_2/g_0 = I_2(SV_1)/I_0(SV_1). \tag{14}$$

With this brief review of mixing theory covered in much greater detail by Torrey and Whitmer [1], the conversion loss of Schottky-barrier diodes will now be explored in terms of their material parameters.

III. Schottky Diodes

A. General Formulation

The intrinsic conversion loss with optimum coupling of a Schottky diode is directly calculable given the area, temperature, and material parameters of the device and the impedance level of its imbedding circuit. The quantity i_0 for both thermionic emitting and tunnel emitting Schottky barriers can be expressed approximately in the following form [8], [9]:

$$i_0 = \tfrac{1}{4}\pi d^2 J_{\max} \exp(-SV_B). \tag{15}$$

With optimum coupling, the RF impedance R_1 can be expressed in terms of V_0, V_1, and the material parameters by setting R_1 equal to R_{10}. From (8), (9), (12)–(15)

$$R_1^{-1} = \tfrac{1}{4}\pi d^2 S J_{\max}(I_0 + I_2)\sqrt{1-\eta}\exp[-S(V_B - V_0)]. \tag{16}$$

Solving this equation for d yields

$$\frac{d}{d_m} = [(I_0 + I_2)\sqrt{1-\eta}]^{-1/2}\exp[\tfrac{1}{2}S(V_B - V_0)] \tag{17}$$

where

$$d_m = (\tfrac{1}{4}\pi R_1 S J_{\max})^{-1/2}. \tag{18}$$

The argument of the modified Bessel functions is SV_1. In (17), d represents the diameter required to yield optimum coupling. Since the expression on the right of (17) is ≥ 1 in the region of interest $V_0 + V_1 \leq V_B$, d_m represents the smallest possible diameter for which the optimum coupling condition can be met. Expressed in terms of operating conditions, d_m is the diameter required to achieve an optimum coupling with the diode biased at flat band with zero LO drive. The constraints on V_0 and V_1 and their relationship to (11) and (17) must now be analyzed.

Constraints on V_0 and V_1 arise from practical considerations as well as from the nature of the emission mechanism. With positive applied voltages, only voltages less than the flat-band value of V_B should be considered since the voltage dependence cannot be neglected as the flat-band ($V \to V_B$) condition is approached. That is, for a Schottky diode, the capacitance C is given by

$$C = \tfrac{1}{4}\pi d^2\left[\frac{q \in N}{2(V_B - V)}\right]^{1/2}, \qquad V < V_B \tag{19}$$

where V is the applied voltage. As the flat-band voltage is approached, the capacitance C approaches very high values leading to excessively long RC charging times.[2] Consequently, a mixer diode with the LO voltage equal to or exceeding V_B would be a low-frequency device.

Constraints on the magnitude of a negative applied voltage depend upon the emission mechanism responsible for conduction. With thermionic emission, the reverse current approaches the saturation value i_0 for small negative voltages as indicated in (3). As the reverse voltage is increased further, the diode goes into a breakdown condition. However, assuming the breakdown voltage is large enough to be ignored, the only constraint on V_1 for a thermionic emitter is from (19)

$$V_1 < V_B - V_0. \tag{20}$$

The substitution of the upper bound $V_1 = V_B - V_0$ of (20) into (17) yields a lower bound on the ratio d/d_m as a function of SV_1 for a thermionic emitting junction. This relationship is plotted in Fig. 2.

When the I–V behavior is dominated by tunnel emission, the junction conducts strongly for both positive and negative applied voltages. For efficient conversion, large negative LO

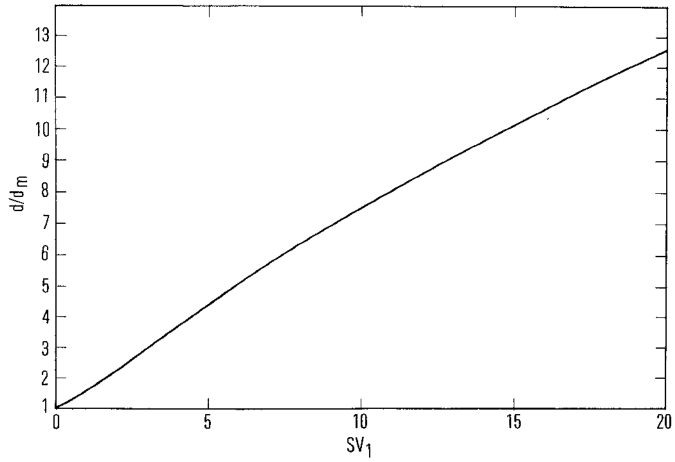

Fig. 2. Normalized diameter versus LO drive for a Schottky-barrier diode mixer under the constraints $V_1 = V_B - V_0$ and $y_1 = y_2 = 1$. This solution is applicable to a thermionic emitter for $V_0 < V_B$ and a tunnel emitter for $\tfrac{1}{2}V_B \leq V_0 < V_B$.

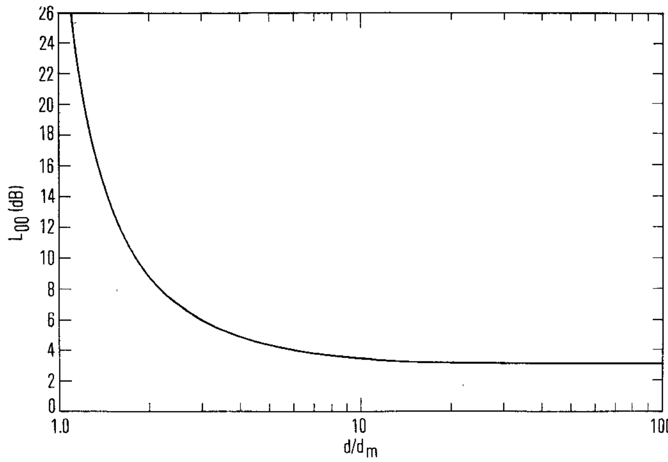

Fig. 3. Intrinsic conversion loss with optimum coupling versus normalized diameter for a Schottky-barrier diode mixer. This solution is applicable to a thermionic emitter for $V_0 < V_B$ and a tunnel emitter for $\tfrac{1}{2}V_B \leq V_0 < V_B$.

currents cannot be tolerated. Hence, to avoid voltage less than zero and greater than V_B, the following restrictions on V_1 are necessary for tunnel emitters:

$$V_1 < V_B - V_0, \qquad \text{for } V_0 \geq \tfrac{1}{2}V_B \tag{21a}$$

$$V_1 < V_0, \qquad \text{for } V_0 \leq \tfrac{1}{2}V_B. \tag{21b}$$

Therefore, the graph shown in Fig. 2 is also suitable as a lower limit on d/d_m for a tunnel emitter if the dc bias is constrained to $V_0 \geq \tfrac{1}{2}V_B$. For $V_0 \leq \tfrac{1}{2}V_B$, (17) and (21b) can be combined in similar fashion to yield a functional relationship between $(d/d_m)\exp(-\tfrac{1}{2}SV_B)$ and SV_1 which is applicable to larger diameters. As will be shown for a tunnel emitter, these constraints on V_0 and V_1 also lead to a minimum in L_{00} as a function of d.

From (11)–(14), (17), and (20) (or in graphical terms, combining Fig. 2 with the well-known plot of L_{00} versus SV_1 that results from (11)–(14) [1]), it is possible to establish a basic relationship between L_{00} and d/d_m for the two emission mechanisms. These relationships are plotted in Fig. 3.[3] The significance

[2] If the diode were being driven by a voltage source, the appropriate R would consist of the series and shunt resistances in parallel.

[3] From the standpoint of rigor, it should be remembered that $V_1 < V_B - V_0$, and hence the values of L_{00} in these and subsequent figures are lower limits on L_{00}.

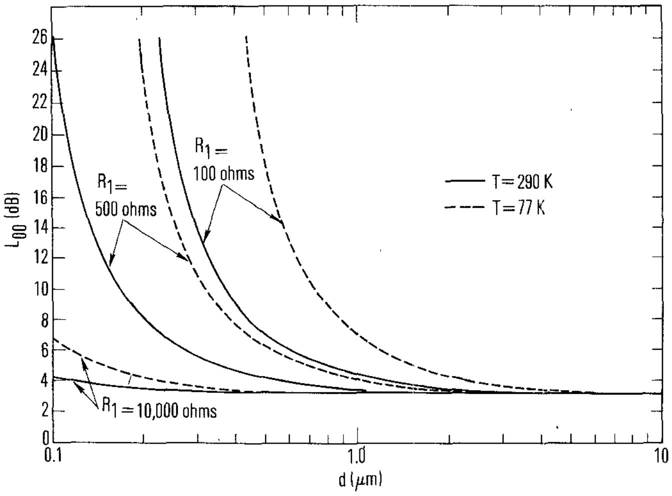

Fig. 4. Intrinsic conversion loss with optimum coupling versus diameter for a thermionic emitting n-type GaAs Schottky-barrier diode at 290 and 77 K with the source impedance as specified and $A^*/A = 0.076$[9]. With $R_1 = 100\ \Omega$ and $T = 290$ K, $d_m = 0.20\ \mu$m.

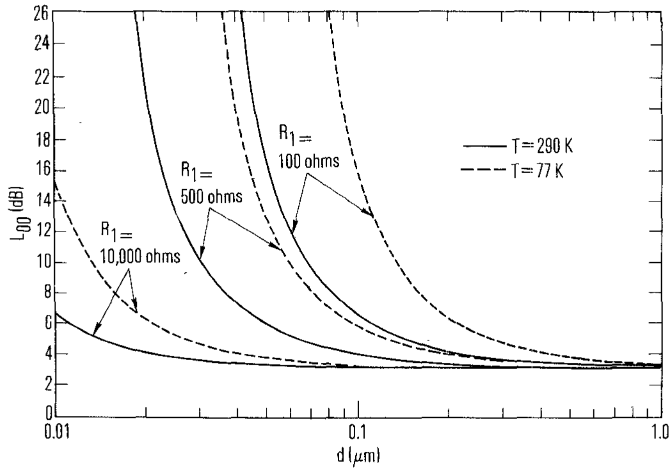

Fig. 5. Intrinsic conversion loss with optimum coupling versus diameter for a thermionic emitting n-type Si Schottky-barrier diode at 290 and 77 K with the source impedances as specified and $A^*/A = 2.2$ [11]. With $R_1 = 100\ \Omega$ and $T = 290$ K, $d_m = 0.038\ \mu$m.

of the diameter d_m now becomes clear. It represents a figure of merit for both the semiconductor under consideration and the circuit to which it must couple. Small values of d_m, and hence large R_1SJ_{max} products, are necessary for good conversion.[4] This formulation will now be applied to specific junctions and circuits.

B. The Thermionic Emitter

The quantities S and J_{max} for a thermionic emitting Schottky barrier are given by [9]

$$S = q/kT \tag{22}$$

and

$$J_{max} = A^*T^2. \tag{23}$$

For free electrons

$$A^* \equiv A = 120\ \text{A/cm}^2\text{K}^2 \tag{24}$$

where A is the Richardson constant for thermionic emission into a vacuum. The value A^* is determined by the band structure of the semiconductor. For n-type GaAs the ratio A^*/A is equal simply to the effective mass ratio m^*/m of the majority carriers [11]. Using the parameters stipulated in (22) and (23), (18) becomes

$$d_m = (\pi q R_1 A^* T/4k)^{-1/2}. \tag{25}$$

Equation (25) indicates that not only are large values of R_1 and A^* desirable, but that reducing the temperature of a thermionic emitting mixer diode will increase its conversion loss even though its nonlinear resistive behavior has greatly increased.

Plots of L_{00} versus d for n-type GaAs Schottky diodes are shown in Fig. 4 for room temperature and liquid nitrogen operation for specific values of RF circuit impedances. The critical nature of temperature, impedance level, and submicron dimensions is in evidence. The marked degradation in conversion with cooling is clearly shown. Although the cooled junction has greater nonlinearity, as given by a larger value of q/kT, the T^2 dependence contained in the term J_{max} overbalances this benefit and produces a larger value of d_m. The purpose of cooling this type of Schottky mixer is, of course, to reduce its noise temperature [12]–[14]. However, Fig. 4 suggests that the reduction in noise could be more than offset by the increase in intrinsic loss if impedances are low and dimensions are small.

Fig. 4 also illustrates the importance of high impedance circuits. An increase in impedance reduces the dc bias current and, hence, reduces the dc voltage which permits a larger level of LO excitation. Unfortunately, high impedance circuitry cannot be readily obtained at millimeter wavelengths, and, as such, an implementation of this approach will depend on future developments. On the other hand, low impedance circuitry is available and tempting to the designer in that it reduces L_p via the third term in (2). The difficulty it can inflict on the intrinsic loss is clearly evident in Fig. 4.

The A^* of n-type silicon, being much larger than that of n-GaAs, extends its usefulness to much smaller diameters. Fig. 5 shows the theoretical conversion loss of an n-type silicon thermionic emitter as a function of d. Its characteristic diameter d_m is smaller than that of n-GaAs by over a factor of 5. From this standpoint, n-Si has a better high-frequency capability than n-GaAs. Of course, in terms of the parasitic loss problem, the opposite situation is true with respect to these materials. However, the distinct advantage n-Si has with its greater current carrying ability suggests the merits of the two materials need further investigation. Multiple contacts [12], [15] can effectively reduce the resistivity of the semiconductor, and, with this technique, n-Si could surpass n-GaAs at very short wavelengths.

C. The Tunnel Emitter

Tunnel emitters have an advantage over thermionic emitters as high-frequency mixers in that they provide a small spreading resistance without having to resort to complex epitaxial structures [16] to reduce R_s. This advantage is somewhat offset by a larger junction capacitance and larger shot noise, which is not reducible with cooling [17].

Both J_{max} and S are only weakly dependent on voltage and for this analysis can be regarded as constants. Using [9, eq. (1)], an evaluation of L_{00} versus d for an n-type GaAs Schottky diode with an electron concentration of 5×10^{18} cm^{-3} is shown in Fig. 6. Since the conduction is a tunneling process, the results

[4] This statement is rigorously true for the conditions stipulated in Fig. 3. The statement breaks down for a tunnel emitter with $V_0 < \frac{1}{2}V_B$ whereupon large values of d_m are desirable. For example, an increase in d_m brought on by a decrease in R_1 would force an increase in V_0 to achieve an impedance match. From (21b) an increase in V_0 permits a corresponding increase in V_1 which would result in an improvement in conversion efficiency.

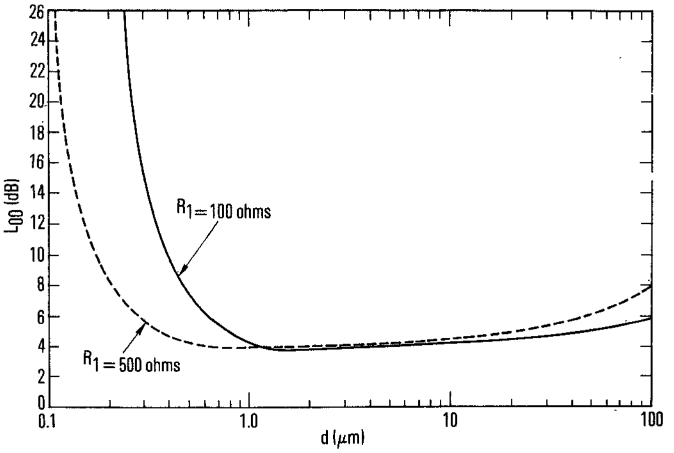

Fig. 6. Intrinsic conversion loss with optimum coupling versus diameter for a tunnel emitting n-type GaAs Schottky-barrier diode with $N = 5 \times 10^{18}$ cm^{-3}, $S = 18$ V^{-1}, $V_B = 0.90$ V, and the source impedances specified.

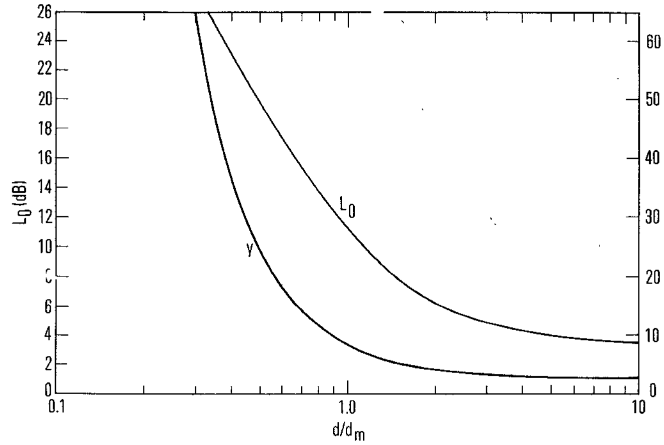

Fig. 7. Intrinsic conversion loss and impedance mismatch ratio versus normalized diameter for a Schottky-barrier diode mixer with $y \equiv y_1 = y_2 \neq 1$. This solution is applicable to a thermionic emitter for $V_0 < V_B$ and a tunnel emitter for $\frac{1}{2}V_B \leq V_0 < V_B$.

are independent of temperature to the degree that V_B and the band gap are temperature independent.

One interesting feature of Fig. 6 is the clear evidence of optimum diameters for conversion. These minima in L_{00}, occurring for the condition $V_1 = V_0 = \frac{1}{2}V_B$, can be reduced further by using larger values of V_B. Hence, as a general feature, large values of V_B are desirable for efficient tunnel emitter converters.

D. Impedance Mismatching

Equation (11) yields the minimum conversion loss for a mixer with given values of V_0 and V_1 having treated the circuit impedances as variable quantities. However, it is far more typical experimentally for V_0 and V_1 to be variables and the circuit impedances to be the constrained quantities. The situation can be explained more clearly in terms of the preceding analysis. Consider a junction with a very small diameter such that a dc bias almost equal to V_B is required to yield the optimum matching condition $y_1 = y_2 = 1$. The quantity SV_1 is so small and the L_{00} versus SV_1 curve so steep under these conditions that it would be far better to relax the $y_1 = y_2 = 1$ restriction by reducing the bias V_0 in order to allow an increase in V_1. As V_0 is reduced still further, the conversion loss would continue to decrease until the increase in the coupling loss due to the impedance mismatch begins to outweigh the benefit of larger V_1. By adjusting V_0 and V_1 in this manner, a minimum in L_0 would be established. A computer solution with $y_1 = y_2$ for this minimum in conversion loss and the resulting mismatch is shown in Fig. 7 as a function of diameter normalized to d_m. Notice that this mode of operation not only achieves smaller conversion losses for a given d but utilizes the region $d < d_m$. It does, however, raise the diode impedance considerably for small d.

At very short wavelengths this solution can lose some of its significance because of the parasitic loss problem. The high diode impedances that result from this mode of operation increase L_p via the third term in (2). Hence at very high frequencies the decrease in L_0 brought on by this solution can be more than offset by the increase in L_p. In general, at all frequencies a mismatch is advantageous but possibly not to the full extent illustrated in Fig. 7. The exact mismatch necessary to achieve a minimum in L_c would depend on the frequency, diode, and circuit in question.

IV. Conclusion

The size, material parameters, and temperature of a Schottky-barrier mixer diode place a lower fundamental limit on its conversion loss in a given circuit. The limit is a fundamental one in that the origin of the limit arises from the mixing process itself and takes on major significance in the design of millimeter and submillimeter wavelength heterodyne receivers. At these frequencies, in order to minimize the effects of the parasitic junction capacitance and series spreading resistance, the junctions must take on micron and submicron dimensions. The analysis presented shows that such dimensions can severely limit the performance of the device irrespective of the loss associated with the parasitic elements. In general, the intrinsic conversion loss is determined by the ratio d/d_m and the impedance matching conditions, as illustrated in Figs. 3 and 7. For thermionic emission, d_m is determined by the Richardson constant of the semiconductor, the temperature of the junction, and the impedance level of the circuit. The larger the product of these three quantities, the more efficient will be the converter.

Note Added in Proof: The Schottky diode has recently been extended to wavelengths of 70 μm as a mixer and 42 μm as a video detector.

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Mode Classification of a Triangular Ferrite Post for *Y*-Circulator Operation

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***Abstract*—Resonance modes of a triangular ferrite post are classified into those which correspond to each eigen-excitation in order to select operation modes of the *Y* circulator. Resonance frequency splits for two rotational phase eigen-excitations are also discussed.**

Introduction

Resonance modes of full height as well as partial-height ferrite junction circulators were studied recently [1]–[3]. Among the various resonance modes, special modes, which correspond to the rotational phase eigen-excitations, principally support the circulator operation. Operation modes are those which have no zero- or threefold symmetry around the ferrite post axis for the *Y* circulator. For a cylindrical ferrite post, operation modes can be easily selected, owing to the field distribution simplicity.

The purpose of this short paper is to classify the resonance modes of a triangular ferrite post into those which correspond to each eigen-excitation in order to select operation modes. The resonance frequencies split, which is one of the fundamental parameters for circulator operation, is also discussed under the assumption of a small anisotropy.

Resonance Mode Classification for Each Eigen-Excitation

In a case of a small anisotropy, the fields in the magnetized ferrite post may be given approximately from those of the demagnetized ferrite post, i.e., dielectric post, using a perturbation theory. Circulation occurs with the pairs of resonance modes which are degenerate when the ferrite is demagnetized. Therefore, we consider the fields in the demagnetized ferrite post. Assuming TM modes and the completely open-circuited condition at the side wall of the ferrite post, the fields are given as follows:

$$E_z = \frac{\chi^2}{j\omega\varepsilon} T(x,y) e^{j\beta z} \tag{1}$$

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$$\begin{aligned} T(x,y) = {} & \cos\left[\frac{2\pi}{3b} l\left(\frac{x}{2} + b\right)\right] \cos\left(\frac{\sqrt{3}\pi(m-n)}{9b} y\right) \\ & + \cos\left[\frac{2\pi}{3b} m\left(\frac{x}{2} + b\right)\right] \cos\left(\frac{\sqrt{3}\pi(n-l)}{9b} y\right) \\ & + \cos\left[\frac{2\pi}{3b} n\left(\frac{x}{2} + b\right)\right] \cos\left(\frac{\sqrt{3}\pi(l-m)}{9b} y\right) \end{aligned} \tag{2}$$

$$H_x = -j \frac{\omega\varepsilon}{\omega^2\varepsilon\mu - \beta^2} \frac{\partial E_z}{\partial y} \tag{3}$$

$$H_y = j \frac{\omega\varepsilon}{\omega^2\varepsilon\mu - \beta^2} \frac{\partial E_z}{\partial x} \tag{4}$$

$$H_z = 0 \tag{5}$$

$$E_x = -j \frac{\beta}{\omega\varepsilon} H_y \tag{6}$$

$$E_y = j \frac{\beta}{\omega\varepsilon} H_x \tag{7}$$

where b is the radius of the inscribed circle of the triangle, l, m, and n are integers which never take zero simultaneously and are related by the following equations:

$$l + m + n = 0 \tag{8}$$

and

$$\chi^2 = \left(\frac{4\pi}{3a}\right)^2 (m^2 + mn + n^2) \tag{9}$$

where a is the side length of the triangle, i.e.,

$$a = 2\sqrt{3}b.$$

These fields are represented by a standing wave in the x–y plane, since they are obtained from those of the triangular metal waveguide [4] by the duality concept. The fields corresponding to the eigen-excitations are traveling or rotating waves in the x–y plane, which take the following phase relation at the ports:

$$\phi_1 = \frac{1}{3}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \phi_2 = \frac{1}{3}\begin{pmatrix}1\\ e^{j(2/3)\pi}\\ e^{-j(2/3)\pi}\end{pmatrix} \qquad \phi_3 = \frac{1}{3}\begin{pmatrix}1\\ e^{-j(2/3)\pi}\\ e^{j(2/3)\pi}\end{pmatrix} \tag{10}$$

where ϕ_1, ϕ_2, and ϕ_3 are the in-phase, clockwise, and counterclockwise rotational phase eigen-excitations, respectively. To classify the resonance modes, it is necessary to obtain fields which correspond to each eigen-excitation.

The fields given by (1)–(7) can be considered to be excited through one of the ports. It is assumed that the port is port 1, for convenience sake (see Fig. 1). The fields excited through other ports are given by the coordinate transformation

$$x \to -\frac{1}{2}x \mp \frac{\sqrt{3}}{2}y \tag{11}$$

$$y \to \pm\frac{\sqrt{3}}{2}x - \frac{1}{2}y \tag{12}$$

where the upper and lower signs denote the cases where excitation is through port 2 and port 3, respectively. Therefore, field